Introduction to IET

Octave Lacourte

ICJ (Lyon)

20-25 Mai

Table of Contents

What means IET

Definition

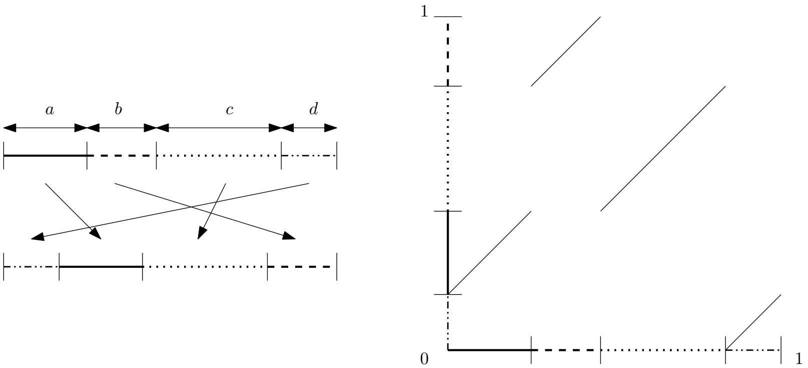
An *interval exchange transformation* is a permutation f of $[0,1[$, continuous outside a finite set and such that for every $x \in [0,1[$ there exists ϵ such that f is a translation on $[x, x+\epsilon[$.

We denote by IET the set of all interval exchange transformation.

What means IET

Definition

An *interval exchange transformation* is a permutation f of $[0,1[$, continuous outside a finite set and such that for every $x \in [0,1[$ there exists ϵ such that f is a translation on $[x, x+\epsilon[$.
We denote by IET the set of all interval exchange transformation.



What means IET

Theorem

$(\mathrm{IET}, \circ)$ *is a group.*

A generating set

Definition

For every a and b in $[0,1[$ with $0 \leq a+b \leq 1$ we called a *restricted-rotation of type* (a,b) every elements f in IET which exchanges two consecutive intervals of length respectively a and b.

A generating set

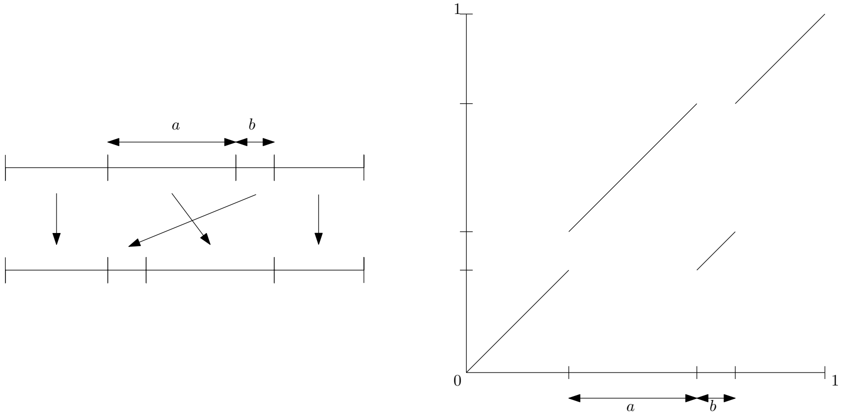
Definition

For every a and b in $[0, 1[$ with $0 \leq a + b \leq 1$ we called a *restricted-rotation of type* (a, b) every elements f in IET which exchanges two consecutive intervals of length respectively a and b.

Definition

We called a *restricted-rotation* every restricted-rotation of type (a, b) for any $0 \leq a, b \leq 1$ with $0 \leq a + b \leq 1$.

A generating set



a restricted rotation of type (a, b)

A generating set

Theorem

The set of all restricted rotation is a generating set for IET.

About its abelianization

Lemma (Arnoux-Fathi-Sah 1981)

The application :

$$\phi : \begin{cases} \text{IET} & \longrightarrow & \mathbb{R} \otimes_{\mathbb{Q}} \mathbb{R} \\ f & \longmapsto & \sum_{a \in \mathbb{R}} a \otimes \lambda((f - id)^{-1}(\{a\})) \end{cases}$$

is a morphism.

About its abelianization

Lemma (Arnoux-Fathi-Sah 1981)

The application :

$$\phi : \begin{cases} \text{IET} & \longrightarrow & \mathbb{R} \otimes_{\mathbb{Q}} \mathbb{R} \\ f & \longmapsto & \sum_{a \in \mathbb{R}} a \otimes \lambda((f - id)^{-1}(\{a\})) \end{cases}$$

is a morphism.

Example

For every a, b in $[0,1]$ with $0 \leq a + b \leq 1$ and every R a restricted-rotation of type (a, b), we have :

$$\phi(R) = b \otimes a - a \otimes b = 2b \wedge a$$

About its abelianization

Corollary

The morphism ϕ is surjective on $\mathbb{R} \wedge_{\mathbb{Q}} \mathbb{R}$.

About its abelianization

Corollary

The morphism ϕ is surjective on $\mathbb{R} \wedge_{\mathbb{Q}} \mathbb{R}$.

Remark

As $\mathbb{R} \wedge_{\mathbb{Q}} \mathbb{R}$ is an abelian group we deduce that $D(\text{IET})$ is in $\text{Ker}(\phi)$.

About its abelianization

Corollary

The morphism ϕ is surjective on $\mathbb{R} \wedge_{\mathbb{Q}} \mathbb{R}$.

Remark

As $\mathbb{R} \wedge_{\mathbb{Q}} \mathbb{R}$ is an abelian group we deduce that $D(\mathrm{IET})$ is in $\mathrm{Ker}(\phi)$.

Theorem

The induced morphism $\Phi : {}^{\mathrm{IET}}\!/_{D(\mathrm{IET})} \longrightarrow \mathbb{R} \wedge_{\mathbb{Q}} \mathbb{R}$ is an isomorphism.

Finding a lamplighter

Definition

The lamplighter group is the restricted wreath product $\mathbb{Z}_2 \wr \mathbb{Z}$.

Finding a lamplighter

Definition

The lamplighter group is the restricted wreath product $\mathbb{Z}_2 \wr \mathbb{Z}$.

Proposition

A presentation of the lamplighter is $< x, t \mid x^2, (xt^{-n}xt^n)^2 >$.

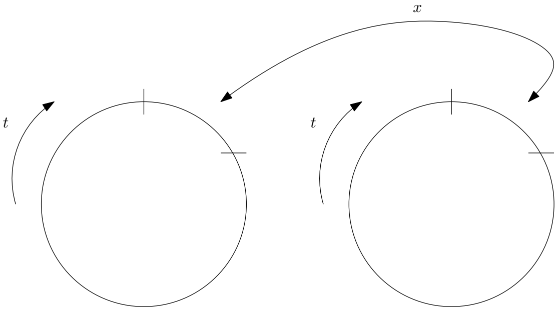
Finding a lamplighter

Definition

The lamplighter group is the restricted wreath product $\mathbb{Z}_2 \wr \mathbb{Z}$.

Proposition

A presentation of the lamplighter is $< x, t \mid x^2, (xt^{-n}xt^n)^2 >$.



François Dahmani, Koji Fujiwara, and Vincent Guirardel.
Solvable groups of interval exchange transformations.
January 2017.

M.Keane.
Interval exchange transformations.
Math. Z., 1975.

P.Arnoux.
Echanges d'intervalles et flots sur les surfaces.
Semin. Les Plans-sur-Bex 1980,, 1981.

Chih-Han Sah.
Scissors congruences of the interval.
preprint, 1981.

Thanks for your attention!